Reliability-based design of roofs exposed to a snow load

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ABSTRACT: Collapses of a number of roofs during the winter period 2005/2006 initiated discussions concerning reliability of the roofs exposed to permanent load and snow. The submitted paper provides a critical analysis of the present design procedures accepted in Eurocodes taking into account available measurements of snow loads in the Czech Republic. In a parametric study the load ratio defined as the characteristic value of the snow load over the sum of characteristic values of the permanent and snow loads is considered. A generic steel structural member is designed using alternative sets of partial factors and reliability of the members is verified by a probabilistic analysis. It appears that the design based on the recommended partial factors may not guarantee an adequate reliability level, in particular for lightweight roofs. The recommended partial factors and design based on the approximate sensitivity factors yield a non-uniform reliability level. The reliability-based design using actual sensitivity factors or the design using a proposed modified partial factor for snow load, dependent on the load ratio, provides a well-balanced reliability level.

1 INTRODUCTION

Collapses of a considerable number of roofs during the winter period 2005/2006 initiated discussions concerning reliability of the roofs exposed to a permanent load and snow. In some countries available measurements of snow loads have been newly evaluated and relevant standards promptly revised. Newly developed maps of snow loads are based on principles of European standards – Eurocodes, specifying the characteristic value of the snow load as the 98% fractile of annual extremes (50-years return period). The design value of the snow load is then determined in accordance with CEN (2002a), using the partial factor 1.5. Alternatively the design can be accomplished considering a target reliability index and sensitivity factor for the snow load approximated by the recommended value -0.7.

The submitted paper provides a critical analysis of the present design procedures accepted in Eurocodes taking into account available measurements of snow loads in the Czech Republic. In a parametric study the load ratio defined as the characteristic value of the snow load over the sum of characteristic values of the permanent and snow loads is taken into account. A generic structural member is designed considering:

- The recommended values of partial factors,
- The recommended approximate values of sensitivity factors and a required reliability level,
- A modified partial factor for the snow load dependent on the load ratio,
- Actual sensitivity factors estimated by FORM and a required reliability level.

Reliability of the generic members is verified by a probabilistic analysis using theoretical models for the basic variables provided by JCSS (2005).

2 PARTIAL FACTOR DESIGN

2.1 *Design based on recommended values of partial factors*

In accordance with the principles of the present suite of the Eurocodes, CEN (2002a), CEN (2002b) and CEN (2004), the characteristic value of the snow load on the ground s_k is specified as the 0.98 fractile of annual extremes (50-year return period). The characteristic load on the roof is then determined as:

$$s_{s,k} = \mu \, C_e \, C_t \, s_k \tag{1}$$

where μ = shape factor (for horizontal roofs equal to 0.8); C_e = exposure factor; and C_t = thermal factor.

The exposure and thermal factors are usually considered as unity, CEN (2004), and hence are omitted in the following analysis. Design of a steel structural member exposed to a permanent load G and snow load S can be based on the partial factor method described in CEN (2002a). Using the fundamental load combination (6.10), the design value of a generic resistance R of the member is determined from:

$$r_k / \gamma_{M0} = \gamma_G\, g_k + \gamma_Q\, s_{s,k} \quad (2)$$

where r_k = characteristic value of resistance; γ_{M0} = partial factor for resistance of a cross-section; γ_G = partial factor for the permanent load; g_k = characteristic value of the permanent load (equal to the mean value); and γ_Q = partial factor for the snow load.

For steel members not susceptible to stability phenomena, the partial factor for resistance is considered by the value 1.0 as recommended in CEN (2005). Assuming unfavourable effects of the actions, the partial factor for the permanent load is 1.35 and the partial factor for the snow load 1.5, CEN (2002a). Design based on these recommended values of the partial factors is referred to as "recommended γ's".

2.2 *Design based on recommended values of sensitivity factors*

Alternatively, the partial factors for the basic variables can be obtained from design values estimated as fractiles of actual probability distributions. According to Annex C of CEN (2002a), the partial factors are determined considering recommended values of sensitivity factors and a target reliability level:

$$\gamma_{M0} = \gamma_{Rd}\, r_k / r_d = 1.05\exp(-2V_R) / \exp(-\alpha_R\, \beta_t\ \ V_R) =$$
$$= 1.05\exp(-2\times0.08) / \exp(-0.8\times3.8\times0.08) = 1.14 \quad (3)$$

$$\gamma_{Qa} = \gamma_{Sd}\, s_{s,d(a)} / (\mu \times s_k) =$$
$$= 1.05 \times 1.61 / (0.8 \times 1) = 2.11 \quad (4a)$$
$$\gamma_{Qb} = \gamma_{Sd}\, s_{s,d(b)} / (\mu \times s_k) =$$
$$= 1.05 \times 1.02 / (0.8 \times 1) = 1.33 \quad (4b)$$

where γ_{Rd} = partial factor for model uncertainty in structural resistance approximated by the value 1.05; r_d = design value of resistance; V_R = coefficient of variation of resistance indicated in Table 1; α_R = FORM sensitivity factor (considered by the recommended value 0.8 for resistance variables, CEN (2002a)); β_t = target reliability index (3.8 for the reference period of 50 years and Class RC2, CEN (2002a)); γ_{Sd} = partial factor for model uncertainty in load effect taken as 1.05, JCSS (1996); and $s_{s,d}$ = design value of the snow load on the roof obtained from (5a,b).

Note that equation (3) is based on the assumption of a lognormal distribution with the lower bound at the origin. In accordance with CNI (2006), the characteristic value of the snow load on the ground 1.0 kN/m^2 is given in the snow map of the Czech Republic for a vicinity of Prague, which may be considered as a typical lowland area in the Czech Republic. The design value of the snow load on the roof is obtained from:

$$\Phi(-\alpha_E\, \beta_t) = \Phi(0.7 \times 3.8) =$$
$$= 1 - 3.9 \times 10^{-3} = P(\mu\, S_{50} < s_{s,d(a)}) \quad (5a)$$
$$\Phi(-0.4\alpha_E\, \beta_t) = \Phi(0.4 \times 0.7 \times 3.8) =$$
$$= 0.86 = P(\mu\, S_{50} < s_{s,d(b)}) \quad (5b)$$

where Φ = cumulative distribution function of the standardized normal variable; α_E = FORM sensitivity factor; and S_{50} = 50-year maxima of the snow load on the ground (the reference period of 50 years is consistent with that assumed for the target reliability index). In accordance with CEN (2002a) the sensitivity factors are approximated by the conservative values -0.7 and -0.4 × 0.7 for the leading action and for accompanying actions, respectively.

Equations (4a) and (5a) apply when the snow load is the leading action:

$$\gamma_{Gb}\, g_k + \gamma_{Qa}\, s_{s,k} > \ \gamma_{Ga}\, g_k + \gamma_{Qb}\, s_{s,k} \quad (6)$$

Otherwise equations (4b) and (5b) are used. The partial factors for the permanent load are obtained using similar relationships ($\gamma_{Ga} \approx 1.35$ for the leading action and $\gamma_{Gb} \approx 1.15$ for an accompanying action). Probabilistic models for the permanent action, shape coefficient and 50-year maxima of the snow load on the ground are given in Table 1. This alternative based on the recommended values of the sensitivity factors and a target reliability level is further referred to as "recommended α's".

2.3 *Design based on partial factor for snow load dependent on a load ratio*

A new approach to determination of the partial factor for snow load is proposed as a quantity dependent on a load ratio χ (similarly as suggested in recent studies by Holicky (2005) and Holicky & Retief (2005) for partial factors of variable actions):

$$\gamma_Q = \gamma_{Sd}\,(1 + \chi) \quad (7)$$

The load ratio is given as the fraction of the characteristic value of the snow load on the roof over the total characteristic load:

$$\chi = s_{s,k} / (g_k + s_{s,k}) \quad (8)$$

A realistic range of the load ratio is from 0.2 to 0.6 as indicated by Gulvanessian & Holicky (2005). However, the ratio may increase up to 0.8 in some cases (e.g. for lightweight steel roofs). For a given load ratio and the characteristic load on the roof, the characteristic permanent load follows from relationship (8) as:

$$g_k = s_{s,k}\,(1 - \chi) / \chi \quad (9)$$

This approach is further referred to as "χ-dependent factor".

2.4 *Reliability-based design*

In this approach the partial factors for the basic variables are derived from actual values of sensitivity factors and a target reliability level:

$$\gamma_{Ri} = r_{k,i} / F_{Ri}^{-1}[\Phi(-\alpha_{Ri} \times \beta_t)]$$
$$\gamma_{Ei} = F_{Ei}^{-1}[\Phi(-\alpha_{Ei} \times \beta_t)] / e_{k,i} \quad (10)$$

where $F^{-1}(\cdot)$ = inverse cumulative distribution function; $r_{k,i}$ = characteristic value of a resistance variable; and $e_{k,i}$ = characteristic value of a load effect variable. The sensitivity factors are obtained by FORM, Hasofer & Lind (1974). The target reliability index is again 3.8. This alternative of design is referred to as "reliability-based design".

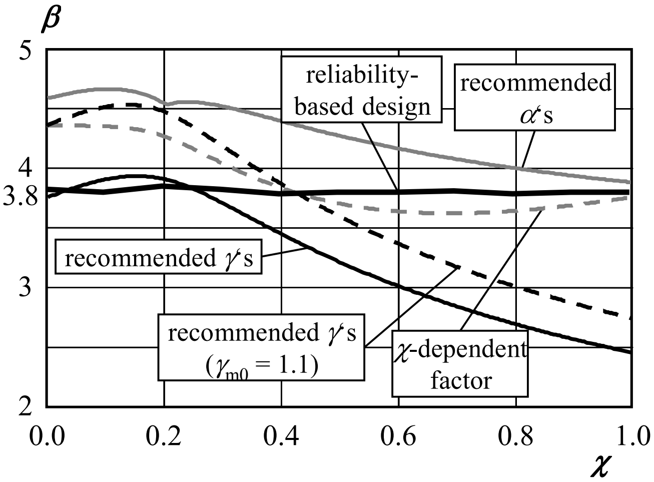

Figure 1. Variation of the reliability index with the load ratio.

3 PROBABILISTIC MODELS FOR BASIC VARIABLES

Reliability of generic steel members designed using the partial factors given above is analysed by probabilistic methods. The limit state function is written as follows:

$$g(\mathbf{X}) = K_R R - K_E (G + \mu S_{50}) \quad (11)$$

where K_R = model uncertainties in structural resistance; and K_E = model uncertainties in load effect. Models for the basic variables are described in Table 1. Note that in the column "partial factors", values considered in the design alternatives "recommended γ's", "recommended α's" and "χ-dependent factor" are indicated, respectively.

Resistance of generic steel members is described by the lognormal distribution with the lower bound at the origin. The coefficient of variation is considered by the realistic value 0.08, JCSS (2005). The mean of resistance is obtained as 1.17-times the characteristic value, which is in accordance with the findings of statistical evaluation of properties of structural steel produced in the Czech Republic reported by Mrazik (1977), Mrazik (1987) and Holicky & Vorlicek (1996). Note that the partial factor for resistance 1.1 accepted from CEN (1992) and Gulvanessian & Holicky (2005) is used in the approach "χ-dependent factor".

Table 1. Models for basic variables.

Variable	Dist.	Partial factors	X_k	μ_X	V_X
Resistance R	LN*	1.0/1.14/1.1	Eq. (2)	$r_k e^{2V_R}$	0.08
Perm. load G	N†	1.35/1.35-1.15/1.35	Eq. (9)	g_k	0.10
Shape coef. μ	N	-	0.8	0.8	0.15
Snow on gr. S_{50}	GU‡	1.5/2.11-1.33/Eq.(7)	s_k	s_k	0.22
Resist. unc. K_R	LN	1.05	-	1.15	0.05
Load ef. unc. K_E	LN	1.05	-	1.0	0.10

*Lognormal distribution with the lower bound at the origin; †Normal distribution; ‡Gumbel distribution of maximum values.

The shape coefficient for horizontal roofs is assumed to be normally distributed. The mean 0.8 derived from the wind speed averaged over a week and the coefficient of variation 0.15 are taken into account according to JCSS (2005).

Measurements of the snow load on the ground provided by the Czech Hydrometeorological Institute for the area of Prague were statistically evaluated in the recent study by Holicky et al. (2007). Assuming the Gumbel distribution for annual maxima of the snow load on the ground, it appears that the mean of the 50-year maxima is approximately the characteristic value given in CNI (2006) while the coefficient of variation is about 0.22. The characteristic value in the new map of snow loads corresponds well to that derived from the measurements. More details are provided by Holicky et al. (2007).

The model uncertainties are described by the lognormal distribution, JCSS (2005). Assuming rolled sections subject to bending about a strong axis when no stability phenomena are taken into account, the mean 1.15 and the coefficient of variation 0.05 of the model uncertainties for resistance follow from evaluation of a number of tests reported in the background document of the Eurocode 3 Editorial Group (1989). The statistical properties of the model uncertainties in load effect are considered in accordance with JCSS (2005).

4 RESULTS OF RELIABILITY ANALYSIS

Results of the reliability analysis are indicated in Figures 1, 2 and 3. Figure 1 shows variation of the reliability index with the load ratio for the different design alternatives. It follows that the recommended values of the partial factors lead to a significant variation of the reliability index. Moreover, for the load ratio greater than 0.3 the index decreases below the target value 3.8 and the reliability of a structural member is insufficient. An acceptable reliability level is achieved only for members exposed to a dominant permanent load ($\chi < 0.3$).

A higher reliability level is provided using the recommended values of the partial factors for the actions and the increased partial factor for resistance ($\gamma_{M0} = 1.1$). However, the reliability level is still inadequate for the load ratio greater than 0.4.

A well-balanced reliability level is achieved for the proposed partial factor of the snow load dependent on the load ratio, which is consistent with results obtained by Holicky et al. (2007).

The design procedure based on the recommended values of the sensitivity factors and the target reliability yields a sufficient reliability level for the whole range of the load ratio. For the load ratio lower than 0.5, this procedure, however, leads to a rather conservative design as the reliability index increases up to 4.2.

The target reliability level is achieved for the whole range of the load ratio using the partial factors based on actual sensitivity factors (and not on the recommended conservative values).

Variation of the sensitivity factors with the load ratio for the design procedure "recommended α's" is shown in Figure 2. It follows that the sensitivity factors are considerably dependent on the load ratio.

The partial factors of the resistance, permanent load and snow load for the "reliability-based design" are derived as follows:

$$\gamma_{M0} = \gamma_{Rd} \times \gamma_{m0}; \quad \gamma_G = \gamma_{Sd} \times \gamma_g;$$
$$\gamma_Q = \gamma_{Sd} \times \gamma_\mu \times \gamma_{S50} \tag{12}$$

The partial factors of basic variables are obtained from relationships (10).

Figure 3 indicates variation of the derived partial factors with the load ratio. It follows that for the considered probabilistic models of the resistance and model uncertainties of resistance, the partial factor γ_M is close to the recommended value 1.0. The partial factor for the permanent load γ_G varies in the range from 1.1 to 1.5.

Significant differences between the recommended value and values derived from the actual sensitivity factors are observed particularly for the partial factor of the snow load γ_Q. The derived partial factor is greater than the recommended value 1.5 nearly for the whole range of the load ratio. It follows that values of about 2.5 – 3.0 would lead to a sufficient reliability level for the significant snow load when the load ratio is greater than 0.3. These findings are consistent with results obtained from analyses of frames in Germany conducted by Sadovsky (2004a,b) and partly also with the background documentation to the Eurocode on basis of structural design published by JCSS (1996). Note that the proposed partial factor for the snow load dependent on the load ratio should also be increased to reach the target reliability level.

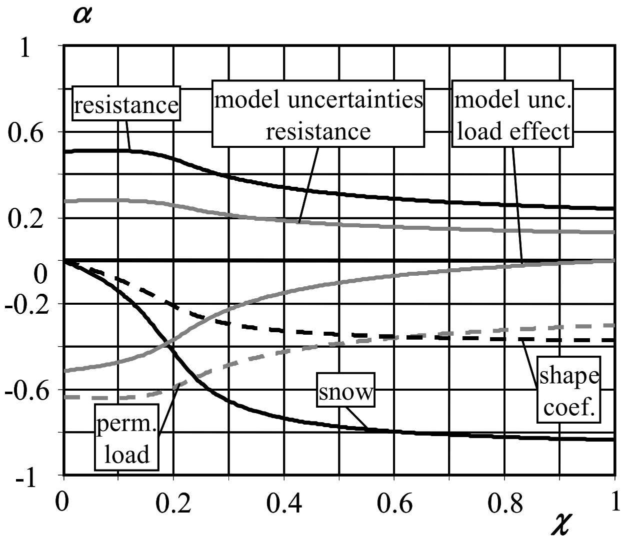

Figure 2. Variation of the sensitivity factors with the load ratio.

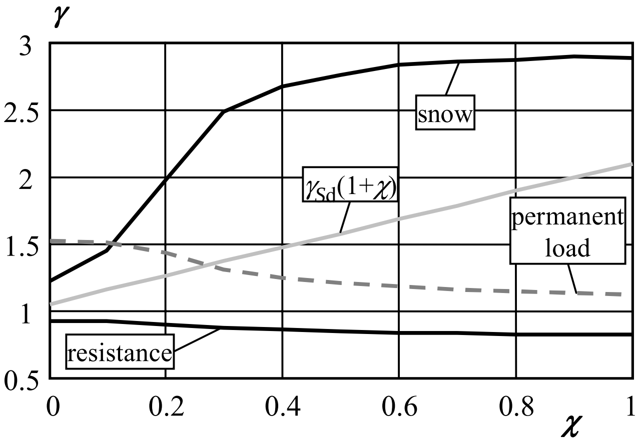

Figure 3. Variation of the partial factors with the load ratio.

It is, however, emphasized that generalization of these findings may be rather difficult. The resulting reliability is considerably dependent on the model uncertainties, which may differ for various types of members or structures under consideration. In addition variability of the snow load effect is significantly increased by uncertainties of the shape coefficient. Formichi (2008) indicated that further research on the shape coefficient is desired.

5 CONCLUDING REMARKS

The following conclusions may be drawn from the presented reliability analysis of steel members exposed to a permanent load and snow load:

- The constant partial factor for the snow load 1.5 leads to a significantly variable (non-uniform) reliability level with respect to the load ratio defined as the characteristic snow load over the total characteristic load.
- For the load ratio greater than 0.3 the reliability index is less than 3.8 and reliability of a structural member is insufficient.

– The partial factor for the snow load should be greater than 1.5 for the load ratio greater than 0.3.
– For the assumed probabilistic models, the partial factor of resistance 1.0 seems to correspond to the partial factor derived from actual sensitivity factors and a target reliability level.
– The target reliability level can be achieved by the reliability-based design where the partial factors for resistance, permanent load and snow load are determined on the basis of sensitivity factors.
– A more uniform reliability level may also be obtained using the partial factor for the snow load dependent on the load ratio.

It is emphasized that the presented results are significantly dependent on the assumed models for basic variables including model uncertainties and should be considered as informative only. Moreover, the snow load model should be further improved.

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REFERENCES

CEN (European Committee for Standardization) 1992. *ENV 1993-1-1 Eurocode 3: Design of Steel Structures - Part 1-1: General - General Rules and Rules for Buildings*. Brussels: CEN.
CEN 2002a. *EN 1990 Eurocode: Basis of structural design*. Brussels: CEN.
CEN 2002b. *EN 1991-1-1 Actions on structures – Part 1-1: General actions – Densities, self weight, imposed loads for buildings*. Brussels: CEN.
CEN 2004. *EN 1991-1-3 Eurocode 1: Actions on Structures – Part 1-3: General actions – Snow loads*. Brussels: CEN.
CEN 2005. *EN 1993-1-1:2005 Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings*. Brussels: CEN.
CNI (Czech Standards Institute) 2006. *CSN EN 1991-1-3 Eurokod 1: Zatizeni konstrukci - Cast 1-3: Obecna zatizeni - Zatizeni snehem, Zmena Z1* (Actions on Structures – Part 1-3: General actions – Snow loads, Revision Z1). Prague: CNI.
Eurocode 3 Editorial Group 1989. *Background Documentation to Eurocode No. 3 Design of Steel Structures Part 1 – General Rules and Rules for Buildings*. Background Document for Chapter 5 of Eurocode 3, Document 5.01.
Formichi, P. 2008. EN 1991 – Eurocode 1: Actions on structures Part 1-3 General actions – Snow Loads. *Handouts of the Workshop EUROCODES Background and Applications*, 18-20 February 2008, Brussels, < http://eurocodes.jrc.ec.europa.eu/>
Gulvanessian, H. & Holicky, M. 2005. Eurocodes: using reliability analysis to combine action effects. *Proceedings of the Institution of Civil Engineers, Structures & Buildings* 158: 243-252.
Hasofer, A.M. & Lind, N.C. 1974. An exact and invariant first order reliability format. *Journal of Engineering Mechanics Division ASCE* 1974, 100(EM1):111–21.
Holicky, M. 2005. Calibration of load combinations for equal safety of concrete members. *Proceedings of the 6th International Congress Global Construction: Ultimate Concrete Opportunities*, 5-7 July 2005, Dundee UK.
Holicky, M., Markova, J. & Sykora M. 2007. Spolehlivost lehkych strech (Reliability of lightweight roofs – in Czech). *Stavebni obzor* (Journal of Civil Engineering) 2007, 16(3): 65-69.
Holicky, M. & Retief, J. 2005. Reliability Assessment of Alternative Eurocode and South African Load Combination Schemes for Structural Design. *Journal of the South African Institution of Civil Engineering* 2005, 47(1): 15-20.
Holicky, M. & Vorlicek, M. 1996. *Zaklady statisticke teorie meze kluzu konstrukcni oceli* (Basis of Statistical Theory of Yield Strength of Structural Steel – in Czech). Final report of the project GACR 103/95/1437. Prague: Klokner Institute, Czech Technical University in Prague.
JCSS (Joint Committee on Structural Safety) 1996. *Background Document Eurocode 1, Part 1: Basis of Design*, JCSS Working Document.
JCSS 2005. *Probabilistic Model Code*. http://www.jcss.ethz.ch/
Mrazik, A. 1977. Statisticke zhodnotenie mechanickych vlastnosti konstrukcnich oceli vyrabanych v rokoch 1971 az 1975 (Statistical Evaluation of Mechanical Properties of Structural Steel Produced from 1971 to 1975 – in Slovak). *Inzenyrske stavby* (Structural Engineering) 9.
Mrazik, A. 1987 *Teoria spolahlivosti ocelovych konstrukcii* (Reliability Theory of Steel Structures – in Slovak). Bratislava: Veda SAV.
Sadovský, Z. 2004a. On optimal reliability-based code calibration – partial factor format. *CTU Reports* 2004, 8(3).
Sadovský, Z. 2004b. On optimal reliability-based code calibration – design value format. *CTU Reports* 2004, 8(3).