Probabilistic prediction of floods and experience from structural failures

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Introduction

Extreme flooding in the Czech Republic in August 2002 affected a number of localities. Emergency measures accepted during the flooding included protective barriers, immediate removing floating debris from bridges, additional anchors of ships on rivers and transport of persons from endangered areas. Despite these measures, damage to construction works was on an unprecedented scale. As indicated in Figure 1, particularly severe consequences were observed in the historic city of Prague where recorded water levels seemed to be exceptionally high.

To reduce consequences of flooding in future, various precautions such as safety barriers and river management including construction of polders, modifications of depth, width and roughness of a river channel are considered. Decisions concerning these expensive measures should be preferably based on cost optimisation. However, such an analysis needs a theoretical model suitable for predicting discharges and extents of future flooding.

In the presented study available hydrological data are evaluated to develop the required model for discharges of the Vltava River in Prague. The methods of moments and of maximum likelihood are applied to analyse the data and to answer frequent questions of civil engineers:

Figure 1. The flooding in Prague and its consequences.

- Was the flooding really so exceptional and unpredictable?
- What was the actual return period corresponding to the observed water level?
- Does the measurement recorded in 2002 affect estimates of characteristic and design values of discharges?

In addition to the statistical analysis of discharges findings of extensive investigations of failure causes are reviewed and recommendations for design and assessment of structures endangered by flooding are outlined.

Statistical evaluation of annual maximum discharges

Annual maximum discharges on the Vltava River in Prague measured by the Czech Hydrometeorological Institute since 1827 are further analysed. Sample characteristics are initially estimated by the classical method of moments described by Ang and Tang (1975) for which prior information on the type of an underlying distribution is not needed.

Point estimates of the characteristics are given in Table 1 for the sample without and with the observation q_{2002}. It appears that the sample mean, standard deviation and coefficient of variation are influenced by the discharge q_{2002} rather insignificantly (the enhancing factor varies in the range from 1.02 up to 1.07). However, the coefficient of skewness seems to be considerably affected by q_{2002} as the enhancing factor is 1.22.

Probabilistic models

The sample characteristics provided in Table 1 indicate that the annual maxima might be described by a two-parameter lognormal distribution having the lower bound at the origin (LN0) or a more general three-parameter shifted lognormal distribution having the lower bound different from zero. Other possible theoretical models are extreme value distributions: the type II called also the Fréchet distribution or type I, a popular Gumbel distribution with the constant skewness 1.14.

Probability density functions of the considered theoretical models and a sample histogram are shown in Figure 2. It appears that the lognormal distribution LN0 describes well the data.

Table 1. Sample characteristics of the annual maxima (sample size n = 165 or 166).

Sample characteristic	Formulae used in the analysis	Without q_{2002}	With q_{2002}	Enhancing factor
Mean	$m = \frac{1}{n}\sum_{i=1}^{n} q_i$	1200 m^3/s	1220 m^3/s	1.02
Standard deviation	$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(q_i - m)^2}$	790 m^3/s	850 m^3/s	1.07
Coefficient of variation	$v = \frac{s}{m}$	0.66	0.69	1.05
Coefficient of skewness	$w = \frac{n}{(n-1)(n-2)s^3}\sum_{i=1}^{n}(q_i - m)^3$	1.43	1.74	1.22

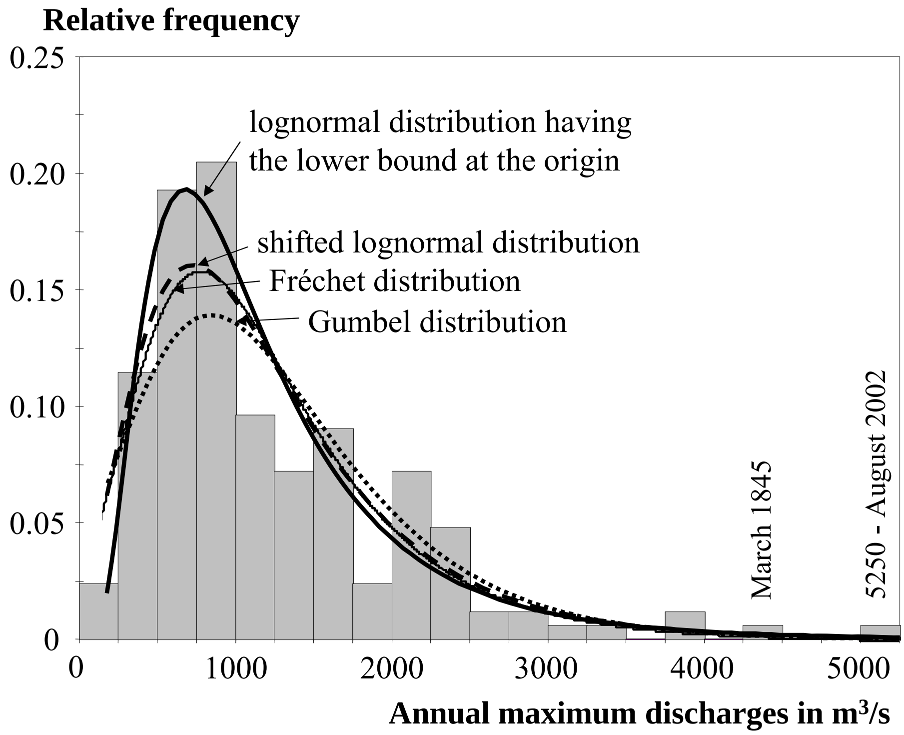

Figure 2. Histogram of the annual maxima and the selected probabilistic distributions.

To compare goodness of fit of the considered distributions, Kolmogorov-Smirnov and chi-square tests described by Ang and Tang (1975) are further applied. A hypothesis that a theoretical distribution fits well a sample distribution should be accepted under the condition

$$K_\mathrm{r} = K_0 / K_p \leq 1 \ (\chi_\mathrm{r}^2 = \chi_0^2 / \chi_p^2 \leq 1) \tag{1}$$

Otherwise the hypothesis should be rejected. In equation (1) K_0 denotes a test value; K_p critical value and K_r relative test value of the Kolmogorov-Smirnov test. Analogous symbols are used for the chi-square test.

Relative test values are listed in Table 2 for the sample without and with q_{2002}. It can be seen that all the applied distributions meet the condition (1) in accordance with the Kolmogorov-Smirnov test. However, the chi-square test indicates that the measured frequencies significantly differ from the theoretical values for all the considered distributions. It appears that the lognormal distribution LN0 is the most suitable model. Less favourable test results are observed for the three-parameter shifted lognormal and Fréchet distribution and the worst test results are obtained for the Gumbel distribution as the fixed skewness of 1.14 may be rather small. If the discharge q_{2002} is involved, the tests provide more favourable results for all the distributions, except for the Gumbel distribution.

Table 2. Results of the Kolmogorov-Smirnov and chi-square tests.

Probabilistic distribution	Without q_{2002}		With q_{2002}	
	K_r	χ_r^2	K_r	χ_r^2
Lognormal distribution LN0	0.53	1.11	0.49	1.01
Shifted lognormal distribution	0.73	1.26	0.65	1.16
Fréchet distribution	0.82	1.35	0.73	1.30
Gumbel distribution	0.85	1.43	0.86	1.63

Table 3. Estimated parameters.

	Without q_{2002}				With q_{2002}			
	m	s	v	w	M	s	v	w
Method of moments	1200	790	0.66	2.3	1220	850	0.70	2.4
Method of maximum likelihood	1210	870	0.72	2.5	1230	910	0.74	2.6

Appropriate models should be selected on the basis of the statistical tests taking into account general experience with distributions of discharges at other localities. Experience of the Czech Hydrometeorological Institute indicates that the lognormal distribution LN0 might be a suitable model. Therefore, this distribution is considered in estimation of extreme discharges only.

Parameter estimation

The method of moments applied to estimate the sample characteristics is often assumed to be not very efficient. Assuming that the underlying distribution of the sample is the lognormal distribution LN0, the sample characteristics can be improved by the method of maximum likelihood, which is considered as the most efficient method for parameter estimation, particularly for large samples. The maximum-likelihood estimators $\hat{m}$ and $\hat{s}$ of unknown mean μ and standard deviation σ of the distribution are obtained maximizing the logarithm of a likelihood function

$$\max_{m,s} \ln[\mathrm{L}(m,s|\mathbf{q})] \rightarrow (\hat{m},\hat{s}) \tag{2}$$

where $\mathbf{q} = (q_1,\ldots,q_n)$ denotes the sample, m and s realization of the parameters μ and σ, respectively, and $\mathrm{L}(m,s|\mathbf{q})$ is the likelihood function

$$\mathrm{L}(m,s|\mathbf{q}) = \prod_{i=1}^{n} \mathrm{f}(q_i|m,s) \tag{3}$$

where f(•) is the probability density function of the underlying distribution. Here the non-linear conjugate-gradient method implemented in the software package Mathcad® is applied.

Comparison of distribution parameters estimated by the method of moments and the method of maximum likelihood is provided in Table 3. It appears that the estimates of the mean are nearly independent of the applied method (differences about 1 %). However, the standard deviations estimated by the method of maximum likelihood are systematically greater than those obtained by the method of moments (differences about 10 %). The assumption of the lognormal distribution LN0 yields the theoretical skewness of about 2.4 while the sample skewness is 1.4 for the sample without q_{2002} and 1.7 for the sample with q_{2002}.

Estimation of extreme values

Upper fractiles q_p of the lognormal distribution LN0 are further estimated using the classical coverage method for a given confidence level γ as indicated e.g. in ISO 12491 (1997)

$$\mathrm{P}(q_{p,\mathrm{cov}} > q_p) = \gamma \tag{4}$$

In accordance with EN 1990 (2002), the characteristic value q_k is obtained as the 0.98 fractile of annual maxima while the design value q_d is the fractile of the life-time maxima corresponding to the probability

$$p_\mathrm{d} = 1 - \Phi(\alpha_E \times \beta) = 1 - \Phi(-0.7 \times 3.8) = 1 - 0.0039 \tag{5}$$

where Φ denotes the cumulative distribution function of the standardised normal distribution, α_E is the FORM sensitivity factor (approximated by the value -0.7 recommended for the leading action) and β is the reliability index equal to 3.8 for a 50-year reference period. Assuming statistical independence of the annual maxima, the design value is estimated as follows

$$q_\mathrm{d} = \mathrm{F}^{-1}\left(\sqrt[50]{p_\mathrm{d}}\right) \tag{6}$$

where $\mathrm{F}^{-1}(\bullet)$ denotes the inverse cumulative distribution function of the underlying distribution of the annual maxima. Partial safety factor γ_Q of discharges is consequently obtained as the ratio $q_\mathrm{d} / q_\mathrm{k}$.

Estimated extreme discharges and partial factors are summarized in Table 4. It is indicated that:

- The extreme values predicted from available data including the discharge q_{2002} are greater than those estimated without this discharge. The differences are by about 5-10 %.
- The extreme discharges predicted by the method of maximum likelihood are greater than those obtained by the method of moments (by about 6-9 % for the characteristic value and 12-19 % for the design value).
- The upper fractiles estimated considering the commonly accepted 0.75 confidence level are greater than the expected upper fractiles (by about 6 % for the characteristic values and 11 % for the design values).
- The partial safety factor $\gamma_Q \approx 3.0$ derived from the data seems to be significantly greater than the recommended value 1.5.

Return period of the discharge recorded in 2002

Expected return periods corresponding to the discharge q_{2002} are derived using the relationship

$$T = 1 / [1 - \mathrm{F}(q_{2002})] \tag{7}$$

where $\mathrm{F}(\bullet)$ denotes the cumulative distribution function of the underlying distribution.

Table 4. Estimated extreme discharges in m^3/s and partial factors.

	Characteristic value				Design value				Partial factor γ_Q
	Without q_{2002}		With q_{2002}		Without q_{2002}		With q_{2002}		
	Expect.	γ= 0.75	Expect.	γ= 0.75	Expect.	γ= 0.75	Expect.	γ= 0.75	
Method of moments	3400	3600	3600	3900	9600	10600	10700	11800	2.8 – 3.1
Method of maximum likelihood	3700	4000	3800	4100	11400	12600	12000	13300	3.1 – 3.3

Table 5. Expected return periods in years.

	Without q_{2002}	With q_{2002}
Method of moments	350	240
Method of maximum likelihood	210	180

Apparently, the periods listed in Table 5 are considerably influenced by the fact whether the discharge q_{2002} is taken into account or not. The estimates based on the method of maximum likelihood are lower than those obtained using the method of moments. Considering the data without q_{2002}, the observed discharge q_{2002} = 5250 m^3/s corresponds to the exceptionally long return period 210 years for the method of maximum likelihood. Obviously this discharge could have been hardly expected. Note that estimates of a return period may enormously vary with a type of the applied distribution as shown by Holicky and Sykora (2004).

Note that the presented study is based on available data only. A more detailed analysis should also consider non-statistical influences that may have evolved during the period covered by the measurements (since 1827). In particular discharges may be strongly dependent on a river management including construction of polders, modifications of depth, width and roughness of a river channel and removal of vegetation. Effects of deforestation and other man-made interventions in environment should also be taken into account.

Causes of structural failures

The main observed causes of structural damage due to the flooding may be subdivided into geotechnical and structural aspects. The geotechnical causes include:

- Insufficient foundation (depth, width),
- Underground transport of sediments and man-made ground (propagation of caverns),
- Increased earth pressure due to elevated underground water.

The major structural causes cover:

- Insufficient structural robustness (no ring beams as indicated in Figure 3),
- Use of inadequate construction materials (unfired masonry units as shown in Figure 4),
- Material property changes caused by moisture (volume, strength).

It has been observed that in particular lack of structural robustness might have led to failures disproportionate to original causes. Structural robustness may be improved by adequate:

- System of horizontal and vertical ties,
- Increased resistance of key members (a member essentially important for the structural robustness in the way that failure of this member implies a failure of a whole structure or significant parts of it),
- Secondary protection of key members,
- Invulnerable structural detailing.

At present robustness is investigated by researchers from more than 20 European countries within the COST Action TU0601 Robustness of structures (www.cost-tu0601.ethz.ch). Preliminary findings are summarized by Faber et al. (2008).

Figure 3. Failure of a structure without ring beams.

Figure 4. Failure of a structure with unfired masonry units.

Conclusions and recommendations for design and assessment of potentially flooded structures

Presented statistical evaluation of annual discharge maxima indicates that:

- Considering results of statistical tests and experience with distributions of discharges in other localities, a lognormal distribution with the lower bound at the origin is a suitable theoretical model for the analysed sample.
- The characteristic and design values of discharges predicted using data including the discharge in 2002 are greater than those estimated without considering this discharge (in most cases by about 5 %).
- Extreme discharges predicted by the method of maximum likelihood are greater than those by the method of moments (by about 10 %).
- The recommended partial safety factor 1.5 is considerably lower than the value derived from the data (approximately 3.0 using the methods of moments and maximum likelihood).
- The discharge observed in 2002 corresponds to an exceptionally long return period and, therefore, could have been hardly expected.

It follows from investigation of structural failures due to the flooding that:

- The main observed causes of structural damage may be subdivided into geotechnical and structural aspects.
- Lack of structural robustness might have led to failures of flooded structures disproportionate to original causes.

Based on the above conclusions, the following recommendations for design and assessment of structures potentially endangered by flooding are provided:

- Prior to decisions concerning safety of flooded structures, available data on discharges should be carefully analysed since extreme discharges predicted from measurements may considerably differ from those provided in standards.
- Robustness aspects should be considered in design to reduce possible damage due to flooding. Sufficient robustness may be achieved by an adequate system of ties, increased resistance of key members, secondary protection of key members and by invulnerable structural detailing.

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