SAFETY OF LIGHTWEIGHT STEEL ROOFS EXPOSED TO SNOW LOAD

Milan Holicky, Miroslav Sykora [a]

[a] Czech Technical University in Prague, Klokner Institute, Prague, Czech Republic

INTRODUCTION

Collapse of a considerable number of lightweight steel roofs in Europe during the winter period 2005/2006 initiated discussions concerning reliability of the roofs exposed to permanent load and snow only. In some countries available measurements of snow loads were newly evaluated and relevant standards were promptly revised. Newly developed maps of snow loads are based on principles of valid European standards (Eurocodes) [1-3] specifying the characteristic value of snow load as the 0.98 fractile of annual extremes (a 50-year return period). The design value of the snow load is then determined using the partial factor 1.5.

The submitted paper provides a critical analysis of present design procedures accepted in Eurocodes. Probabilistic reliability assessment is based on available measurements of snow loads. It appears that in case of lightweight steel roofs the present requirements of available Eurocodes may not guarantee an adequate reliability level. An alternative procedure of safety design of the roofs exposed to permanent and snow load is proposed. It is foreseen that the partial factor for snow load should be increased.

1 PARTIAL FACTOR DESIGN

1.1 Design Based on Recommended Values of Partial Factors

In accordance with the principles of the present suite of European standards [1-3] the characteristic value of the snow load on the ground s_k is specified as the 0.98 fractile of annual extremes (the 50-year return period) [1,2]. The characteristic load on the roof is then determined as

$$s_{s,k} = \mu \, C_e \, C_t \, s_k \tag{1}$$

where μ shape factor (for horizontal roofs equal to 0.8)
C_e exposure factor
C_t thermal factor

The exposure and thermal factors are usually considered as unity [3] (and omitted further on). Design of a steel structural member exposed to a permanent load G and snow load S can be based on the partial factor method [1]. Using the fundamental load combination (6.10), the design value of a generic resistance R of a member is determined from

$$R_k \,/\, \gamma_{M0} = \gamma_G \, G_k + \gamma_Q \, s_{s,k} \tag{2}$$

where R_k characteristic value of resistance
γ_{M0} partial factor for resistance of a cross-section
γ_G partial factor for the permanent load
G_k characteristic value of the permanent load (equal to the mean value)
γ_Q partial factor for the snow load

For steel members not susceptible to stability phenomena, the partial factor for resistance is further considered by the value 1.0 recommended in [4]. Assuming unfavourable effects of the actions, the partial factor for the permanent load is 1.35 and the partial factor for the snow load 1.5 [1]. Design based on these recommended values of the partial factors is referred to as “recommended γ’s”.

1.2 Design Based on Recommended Values of Sensitivity Factors

Alternatively, the partial factors for the basic variables can be obtained from design values estimated as fractiles of probability distributions. Accordance to Annex C [1], the partial factors are determined considering recommended values of sensitivity factors and a target reliability level

$$\gamma_{M0} = \gamma_{Rd}\, R_k / R_d = 1.05 \exp(-2V_R) / \exp(-\alpha_R\, \beta_t\, V_R) =$$
$$= 1.05 \exp(-2 \times 0.08) / \exp(-0.8 \times 3.8 \times 0.08) = 1.14 \quad (3)$$
$$\gamma_{Qa} = \gamma_{Sd}\, s_{s,d(a)} / (\mu \times s_k) = 1.05 \times 1.61 / (0.8 \times 1) = 2.11 \quad (4a)$$
$$\gamma_{Qb} = \gamma_{Sd}\, s_{s,d(b)} / (\mu \times s_k) = 1.05 \times 1.02 / (0.8 \times 1) = 1.33 \quad (4b)$$

where γ_{Rd} partial factor for model uncertainty in structural resistance approximated by the value 1.05
R_d design value of resistance
V_R coefficient of variation of resistance indicated in Table 1
α_R FORM sensitivity factor (considered by the recommended value 0.8 for resistance variables [1])
β_t target reliability index (3.8 for the reference period of 50 years and Class RC2 [1])
γ_{Sd} partial factor for model uncertainty in load effect taken as 1.05 [5]
$s_{s,d}$ design value of the snow load on the roof obtained from *Eqs. (5a,b)*

Note that *Eq. (3)* is based on the assumption of a lognormal distribution with the lower bound at the origin. The characteristic value of the snow load on the ground of 1.0 kN/m^2 is given in the snow map of the Czech Republic for a vicinity of Prague [6], which may be considered as a typical lowland area in the Czech Republic. The design value of the snow load on the roof in *Eq. (4)* is obtained from

$$\Phi(-\alpha_E\, \beta_t) = \Phi(0.7 \times 3.8) = 1 - 3.9 \times 10^{-3} = P(\mu\, S_{50} < s_{s,d(a)}) \quad (5a)$$
$$\Phi(-0.4\alpha_E\, \beta_t) = \Phi(0.4 \times 0.7 \times 3.8) = 0.86 = P(\mu\, S_{50} < s_{s,d(b)}) \quad (5b)$$

where Φ cumulative distribution function of the standardized normal variable
α_E FORM sensitivity factor (-0.7 for the leading action and -0.4 × 0.7 for the accompanying actions [1])
S_{50} 50-year maxima of the snow load on the ground (the reference period of 50 years is consistent with that assumed for the target reliability index β)

Eqs. (4a) and *(5a)* apply when the snow load is the leading action

$$\gamma_{Gb}\, G_k + \gamma_{Qa}\, s_{s,k} > \gamma_{Ga}\, G_k + \gamma_{Qb}\, s_{s,k} \quad (6)$$

Otherwise *Eqs. (4b)* and *(5b)* are used. The partial factors for the permanent load are obtained using similar relationships ($\gamma_{Ga} \approx 1.35$ for the leading action and $\gamma_{Gb} \approx 1.15$ for an accompanying action). Probabilistic models for the permanent action, shape coefficient and 50-year maxima of the snow load on the ground are given in Table 1. This alternative based on the recommended values of the sensitivity factors and a target reliability level is further referred to as "recommended α's".

1.3 Design Based on Partial Factor for Snow Load Dependent on a Load Ratio

Finally a new approach to determination of the partial factor for snow load is proposed as a quantity dependent on a load ratio χ (similarly as suggested in recent studies [7,8] for partial factors of variable actions)

$$\gamma_Q = \gamma_{Sd}\, (1 + \chi) \quad (7)$$

The load ratio is given as the fraction of the characteristic value of the snow load on the roof over the total characteristic load

$$\chi = s_{s,k} / (G_k + s_{s,k}) \quad (8)$$

The load ratio χ for the lightweight steel roofs is expected to vary in the range from 0.4 to 0.8. For a given χ and $s_{s,k}$ the characteristic permanent load follows from *Eq. (8)* as

$$G_k = s_{s,k}\,(1-\chi)\,/\,\chi \tag{9}$$

This approach is referred to as "χ-dependent factor".

2 PROBABILISTIC MODELS FOR BASIC VARIABLES

Reliability of generic steel members designed using the partial factors obtained above is analysed using probabilistic methods. The limit state function is given as

$$g(\mathbf{X}) = K_R\,R - K_E\,(G + \mu\,S_{50}) \tag{10}$$

where K_R model uncertainties in structural resistance
K_E model uncertainties in load effect

Probabilistic models for the basic variables are described in Table 1.

Table 1. Models for basic variables

Variable	Symb.	Distr.	Recommended γ's / recommended α's / χ-dependent factor	Char.val. X_k	Mean μ_X	CoV V_X
Resistance	R	LN*	1.0/1.17/1.1	*Eq. (2)*	$R_k\exp(-2V_R)$	0.08
Permanent load	G	N†	1.35/1.35(1.15‡)/1.35	*Eq. (9)*	G_k	0.10
Shape coefficient	μ	N	-	0.8	0.8	0.15
Snow load on the ground, 50-year max.	S_{50}	GU§	1.5/2.11(1.33‡)/ /1.05(1+χ)	s_k	s_k	0.22
Resistance uncert.	K_R	LN	1.05	-	1.15	0.05
Load effect uncert.	K_E	LN	1.05	-	1.0	0.10

*Lognormal distribution with the lower bound at the origin †Normal distribution ‡Accompanying action §Gumbel distribution of maximum values

Resistance of generic steel members is described by the lognormal distribution with the lower bound at the origin. The coefficient of variation is considered by the realistic value 0.08 [9]. The mean of resistance is obtained as 1.17-times the characteristic value, which is in accordance with the findings of statistical evaluation of properties of structural steel produced in the Czech Republic [10-12]. Note that the partial factor for resistance 1.1 accepted from [13,14] is used in the approach "χ-dependent factor".

The shape coefficient for horizontal roofs is assumed to be normally distributed. The mean 0.8 derived from the wind speed averaged over a week and the coefficient of variation 0.15 are taken into account [9].

Data provided by the Czech Hydrometeorological Institute for the area of Prague are statistically evaluated in [15]. Assuming the Gumbel distribution for annual maxima of the snow load on the ground, it appears that the mean of the 50-year maxima is approximately the characteristic value given in [6] while the coefficient of variation is about 0.22. The characteristic value in the new map of snow loads corresponds well to that obtained from the measurements. More details are provided in [15].

The model uncertainties are described by the lognormal distribution [9]. Assuming rolled sections subject to bending about the strong axis when no stability phenomena are taken into account, the mean 1.15 and the coefficient of variation 0.05 of the model uncertainties for resistance are accepted [16]. The statistical properties of the model uncertainties in load effect are considered in accordance with [9].

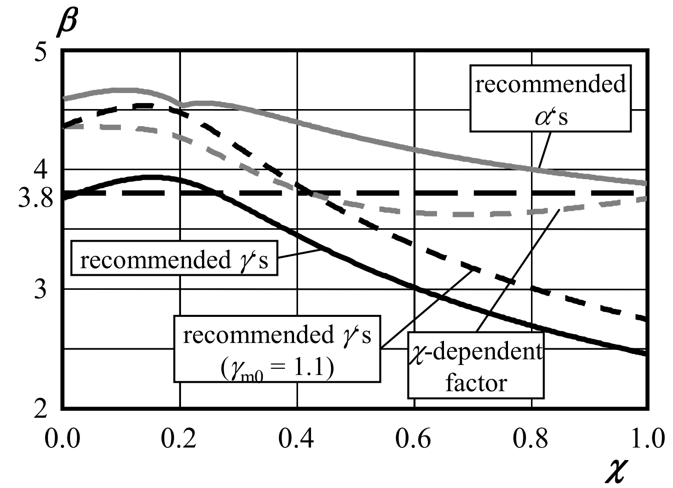

Fig. 1. Variation of the reliability index with the load ratio

3 RESULTS OF RELIABILITY ANALYSIS

Results of the reliability analysis are indicated in Figures 1, 2 and 3. Figure 1 shows variation of the reliability index with the load ratio for the different alternatives of the design.

It follows from Figure 1 that the recommended values of the partial factors lead to a significant variation of the reliability index with the load ratio. Moreover, for the load ratio $\chi > 0.3$ the index decreases below the target value 3.8 recommended in [1] and the reliability of a structural member is insufficient. An acceptable reliability level is achieved only for steel members exposed to a dominant permanent load ($\chi < 0.3$).

A higher reliability level is provided using the recommended values of the partial factors for the actions and the increased partial factor for resistance ($\gamma_{m0} = 1.1$). However, the reliability level is still inadequate for the greater load ratio $\chi > 0.4$.

A well-balanced reliability level is obtained for the proposed partial factor dependent on the load ratio as also follows from previous studies [8,15]. For the lightweight steels roofs ($0.4 < \chi < 0.8$), the resulting reliability is close to the target level.

The design procedure based on the recommended values of the sensitivity factors and the target reliability yields a sufficient reliability level for the whole range of the load ratio. For the load ratio $\chi < 0.5$ this procedure, however, leads to slightly conservative design as the reliability index is greater than 4.2. To achieve the target reliability level for the whole range of the load ratio, the partial factors should be based on actual sensitivity factors and not on the recommended (conservative) values.

Variation of the sensitivity factors with the load ratio for the design procedure "recommended α's" is shown in Figure 2. It follows that the sensitivity factors are considerably dependent on the load ratio and the partial factors derived from the sensitivity factors should be accordingly modified.

Figure 3 indicates variation of the partial factors for the resistance, permanent load and snow load including model uncertainties with the load ratio. Note that variability of the shape coefficient is also considered in the partial factor for the snow load.

It follows from Figure 3 that for the considered probabilistic models of the resistance and model uncertainties of resistance, the partial factor γ_M is close to the recommended value 1.0. The partial factor for the permanent load γ_G varies in the range from 1.1 to 1.5.

Significant differences between the recommended value and values derived from the actual sensitivity factors are observed for the partial factor of the snow load γ_Q. The derived partial factor is greater than the recommended value 1.5 nearly for the whole range of the load ratio. It follows that values of about 2.5 – 3.0 would lead to a sufficient reliability level for the significant snow load ($\chi > 0.3$). These findings are consistent with results obtained from analyses of frames in Germany [17,18] and partly also with the background documentation to Eurocode on basis of design [5].

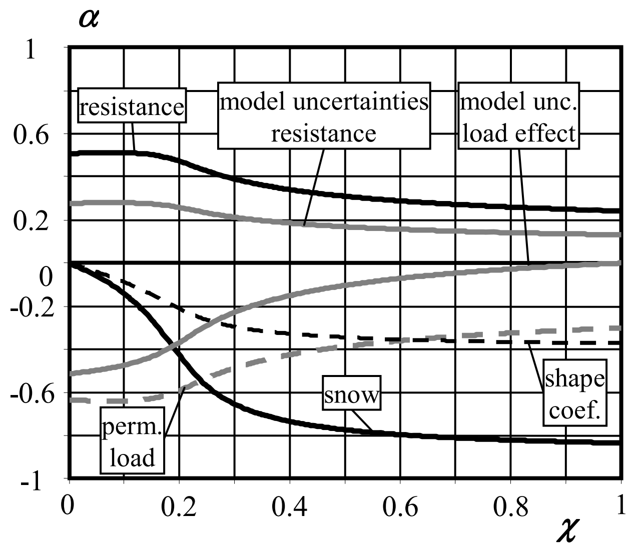

Fig. 2. Variation of the sensitivity factors with the load ratio

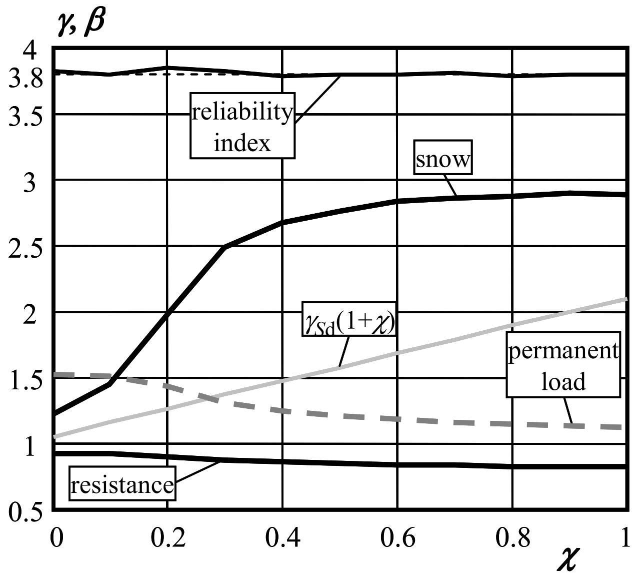

Fig. 3. Variation of the partial factors and the reliability index with the load ratio

It is observed that the proposed partial factor for the snow load dependent on the load ratio should be also increased to reach a sufficient reliability level.

Reliability of the steel members design using the derived factors γ_M, γ_G and γ_Q is indicated in Figure 3. The reliability index is approximately equal to the target value 3.8.

It is, however, emphasized that generalization of the outlined findings may be rather difficult. The resulting reliability is considerably dependent on the model uncertainties, which may differ for various types of members or structures under consideration. In addition variability of the snow load effect is significantly increased by uncertainties of the shape coefficient. Further research on the shape coefficient is desired as concluded in [19].

4 CONCLUDING REMARKS

The following conclusions may be drawn from the presented reliability analysis of steel members exposed to a permanent load and snow load:

1. The constant partial factor for the snow load 1.5 leads to a significantly variable (non-uniform) reliability level with respect to the load ratio χ defined as the characteristic snow load over the total characteristic load.
2. For the load ratio $\chi > 0.3$ the reliability index is less than 3.8 and reliability of a structural member is insufficient.
3. The partial factor for the snow load should be greater than 1.5 for the load ratio $\chi > 0.3$.
4. Considering the probabilistic models assumed for the basic variables in this study (particularly the mean value 1.15 of the resistance uncertainties), the value 1.0 recommended for the partial factor of resistance corresponds to the partial factor derived from actual sensitivity factors and a target reliability level.
5. To reach the target reliability level, the partial factors for resistance, permanent load and snow load should be determined on the basis of actual sensitivity factors and a target reliability level.
6. A more uniform reliability level may also be obtained using the partial factor for the snow load dependent on the load ratio.

It is emphasized that the presented results are significantly dependent on the assumed models for basic variables and should be considered as informative only. In particular the snow load model should be further improved.

ACKNOWLEDGMENT

This study has been conducted in the Klokner Institute, CTU in Prague, in the framework of the research project OC08059 Assessment of Structural Robustness supported by the Ministry of Education, Youth and Sports of the Czech Republic.

REFERENCES

[1] *EN 1990 Eurocode: Basis of structural design*, CEN, 2002

[2] *EN 1991-1-1 Actions on structures – Part 1-1: General actions – Densities, self weight, imposed loads for buildings*, CEN, 2002

[3] *EN 1991-1-3 Eurocode 1: Actions on Structures – Part 1-3: General actions – Snow loads*, CEN, 2004

[4] *EN 1993-1-1:2005 Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings*, CEN, 2005

[5] *Background Document Eurocode 1, Part 1: Basis of Design*, JCSS Working Document, 1996

[6] *ČSN EN 1991-1-3 Eurokód 1: Zatížení konstrukcí - Část 1-3: Obecná zatížení - Zatížení sněhem*, Změna Z1 (Actions on Structures – Part 1-3: General actions – Snow loads, Revision Z1), ČNI, 2006

[7] Holický M., Retief J., Reliability Assessment of Alternative Eurocode and South African Load Combination Schemes for Structural Design, *Journal of the South African Institution of Civil Engineering* 2005, 47(1)

[8] Holický M., Calibration of load combinations for equal safety of concrete members, *Proceedings of the 6th International Congress Global Construction: Ultimate Concrete Opportunities*, 5-7 July 2005, Dundee UK

[9] *JCSS Probabilistic Model Code*, http://www.jcss.ethz.ch/, 2005

[10] Mrázik A., *Teória spoľahlivosti oceľových konštrukcií* (Reliability Theory of Steel Structures – in Slovak), Veda SAV, Bratislava, 1987

[11] Mrázik A., Statistické zhodnotenie mechanických vlastností konstrukčních ocelí vyrábaných v rokoch 1971 az 1975 (Statistical Evaluation of Mechanical Properties of Structural Steel Produced from 1971 to 1975 – in Slovak), *Inženýrské stavby* 9, 1977

[12] Holický M., Vorlíček M., *Základy statistické teorie meze kluzu konstrukční oceli* (Basis of Statistical Theory of Yield Strength of Structural Steel – in Czech). Final report of the project GAČR 103/95/1437, Prague, Klokner Institute, Czech Technical University in Prague, 1996

[13] *ENV 1993-1-1 Eurocode 3: Design of Steel Structures - Part 1-1: General - General Rules and Rules for Buildings*, CEN, 1992

[14] Gulvanessian H., Holicky M., Eurocodes: using reliability analysis to combine action effects, *Proceedings of the Institution of Civil Engineers, Structures & Buildings 158*, 2005

[15] Holicky M., Markova J., Sykora M., Spolehlivost lehkých střech (Reliability of lightweight roofs – in Czech), *Stavební obzor* (Journal of Civil Engineering) 2007, 16(3)

[16] *Background Documentation to Eurocode No. 3 Design of Steel Structures Part 1 – General Rules and Rules for Buildings*, Background Document for Chapter 5 of Eurocode 3, Document 5.01, 1989

[17] Sadovský Z., On optimal reliability-based code calibration – partial factor format, *CTU Reports* 2004, 8(3)

[18] Sadovský Z., On optimal reliability-based code calibration – design value format, *CTU Reports* 2004, 8(3)

[19] Formichi P., EN 1991 – Eurocode 1: Actions on structures Part 1-3 General actions – Snow Loads, Handouts of the Workshop EUROCODES Background and Applications, 18-20 February 2008, Brussels, < http://eurocodes.jrc.ec.europa.eu/>